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Theoretical understanding of the effects of advection schemes and SGS models for LES practical guidelines

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SUMMARY

The rapid growth of computational fluid dynamics (CFD) in wind engineering highlights the need for clear and reliable numerical practice guidelines, especially as large-eddy simulation (LES) becomes recently adopted by non-specialists. This study theoretically examines how advection schemes and sub-grid scale (SGS) models influence LES accuracy using wave propagation theory. Dispersion relation is derived for a one-dimensional constant advection equation to quantify phase velocity and decay factor as functions of wave number and wavelength. The results demonstrate that discretization inherently reduces wave propagation speed and amplifies numerical damping, particularly when the wavelength approaches to the grid spacing, while high-order schemes alleviate these effects. Based on these theoretical derivations, recent results on the effective grid scale of turbulent flows around a block were discussed in the presentation. This work provides an intuitive foundation for selecting numerical schemes and SGS models, supporting the development of reliable best-practice guidelines for the computational wind engineering.

Keywords: *Advection scheme, SGS model, Phase velocity, Decay factor, Large-eddy simulation*

1. INSTRUCTION

The rapid increase in studies related with computational fluid dynamics (CFD) in wind engineering clearly demonstrates the importance of providing precise guidance of the proper use of simulation techniques. The growing demands in CFD simulation along with advances in computational resources inevitably have brought many non-specialists into these fields. To properly disseminate and promote CFD in wind engineers, practitioner, and even designers, establishing guidelines for computational wind engineering (CWE) has been an essential objective. Following establishments of the best practice guidelines for steady simulation based on Reynolds-Averaged Navier–Stokes simulation (AIJ and COST guidelines, Franke et al. 2007, Tominaga et al. 2008), academic and social demands also create a need to develop recommendations for large-eddy simulation (LES).

Among the various factors required to obtain plausible LES results, the LES reliability depends on whether LES concept, resolving only large-scale eddies while modelling the less influential sub-grid scale (SGS) eddies, is adequately satisfied. The LES quality is technically evaluated using the ratio between resolved scale turbulent kinetic energy (TKE) to SGS-TKE. However, more direct and simple indices are desirable, especially for practitioners. In this context, we proposed the effective grid length scale (Tong et al. 2025), defined by the cut-off length scale obtained from power spectrum densities, which differentiates the physical grid size with the resolved eddy size. We believe that the length scale is more intuitive for LES users and easily linked to recommendations for numerical settings. Building on our previous work determining the effective grid length scale, this paper theoretically explains the effect of advection schemes and SGS models based on wave propagation theory.

2. THEORY

Fluid motion for built environments is expressed by the incompressible continuity and Navier–Stokes equations, which hold advection terms representing the transport of physical quantities, and diffusion terms working to reduce the fluctuations in the quantities. To understand wave propagation and attenuation in such velocity fields, an approach is to derive the dispersion relation where angular frequency ω is expressed by wave number k of a general wave

$$\phi = Aexp(i(kx - \omega t)), \tag{1}$$

where A, x, and t represent the amplitude, spatial coordinate, and time. By substituting Eq. (1) into the one-dimensional constant advection–diffusion equation of ϕ, we can obtain the dispersion relation as $\omega(k) = Ck - iDk^2$, where C and D are the advection velocity and diffusivity.

A phase velocity v_p, representing a wave propagation speed with a constant phase is defined as

$$v_p(k) = \frac{Re\{\omega\}}{k}, \tag{2}$$

in which only the real part of ω determines the velocity because the imaginary part does not affect the propagation of the wave.

Meanwhile, by substituting the dispersion relation into Eq. (1), the imaginary part works as a decay term, $exp(-Dk^2t)$, on the wave $exp(ikx)$. The decay term indicates that waves temporally damp since the term is expressed as a function of t. Because a wave with a wave number k (wavelength $\lambda = 2\pi/k$) can be propagated with the phase velocity v_p, we can assume the time scale for the wave to decay as $T = 2\pi/k|v_p|$. Accordingly, a decay factor $d(k)$ is expressed as

$$d(k) = \exp(Im\{\omega\}T) = \exp\left(\frac{2\pi Im\{\omega\}}{k|v_p|}\right) \tag{3}$$

3. RESULTS AND DISCUSSIONS

3.1. Advection scheme

In the same manner of deriving dispersion relation, the effect of discretization on the wave propagation can be discussed (Lele, 1992). When a discrete form of $\phi(x)$ is denoted as $\phi_i = \phi(i\Delta)$ at a node i with a uniform grid spacing Δ, the upwind interpolation scheme (hereafter, Upwind) in a finite volume method approximates the advection term as

$$\frac{\partial \phi}{\partial x}_i = \frac{\phi_i - \phi_{i-1}}{\Delta} = \phi_i \frac{1 - \exp(-ik\Delta)}{\Delta}. \tag{4}$$

Therefore, the dispersion relation of a one-dimensional advection equation can be written as

$$\omega = -C\frac{\sin(k\Delta)}{\Delta} - i\frac{\cos(k\Delta) - 1}{\Delta}. \tag{5}$$

Based on Eqs. (2) and (3), v_p and d are determined as functions of k. Accordingly, we also formulate them for other four advection schemes: Linear, Blend, QUICK, and Linear-Upwind. The Linear is the second-order linear interpolation, Blend use both Linear and Upwind interpolation schemes with a Upwind blending ratio α, QUICK indicates Quadratic Upstream Interpolation for Convective Kinematics, using the second-order quadratic interpolation with ϕ_{i+1}, ϕ_i, ϕ_{i-1}, ϕ_{i-2} and Linear-Upwind scheme also employs the second-order quadratic interpolation with the four surrounding stencils. The details of the advection schemes can be referred in Tong et al. (2025).

Figure 1(a) shows the wave number k (wavelength $\lambda = 2\pi/k$) dependency of v_p for the four schemes. Since v_p is defined by the real part of ω, three schemes of Upwind, Linear, and Blend give the same curve. Although we assume the constant advection equation, the results clearly show that the values of v_p attenuate with the increase of k (or decrease of λ) because of the discretization errors. For the Upwind, Linear, Blend, v_p at $\lambda = 4\Delta$ approximately 0.6, indicating the wave can be propagated with a 40% less velocity in numerical simulation than that given in the original equation. Similarly, the QUICK and Linear-Upwind also show the similar steep reduction of v_p, though they can hold v_p values for shorter wavelengths. In addition, all three functions reach 0 at $\lambda = 2\Delta$. Note that the nodes are arranged in a uniform distance, and the cell face values are not used, but values at nodes are directly used to discretize the gradient of ϕ at i.

Figure 1(b) indicates the same but of d for the four schemes. If the values of d remain near 1.0, it means no decay occurs due to discretization. Note that, when the linear advection equation is considered, the values of d should remain equal to 1.0 for the entire range of k, because there is no diffusion term in the original equation. However, once a discretization scheme is introduced, the results show that dramatic reduction of velocity fluctuations can occur, especially in the range of large wave number (small wavelength). The most significant reduction can be confirmed in the Upwind scheme, in which the decay factor starts from 0.8 at $\lambda = 100\Delta$, monotonically decreases with λ, and reached to 0 near $\lambda = 5\Delta$. The QUICK and Linear-Upwind can sustain d around 1.0 up to $\lambda \sim 10\Delta$, and gradually the values of d decrease with λ. Even using these schemes, d is around 0.5 at $\lambda = 5\Delta$, implying vortices with several grid sizes can also simply attenuate rapidly than reality. The Blend (5%) shows slight decreases in the decay factor from larger wavelength, but it can somehow remain over 0.6 at $\lambda = 3\Delta$, and eventually reaches to 0 at $\lambda = 2\Delta$.

3.2. SGS model – Smagorinsky coefficient

SGS models also work as diffusion terms in general. By assuming that the diffusion term with sub-grid scale eddy viscosity ν_{SGS} can be written as $\nu_{SGS}\partial^2\phi/\partial x^2$, we can consider qualitatively how the SGS term change the decay factor using this simplification. The term gives the dispersion relation, v_p and d in the same procedure. The standard Smagorinsky model, formulated as $\nu_{SGS} = C_s^2\Delta q$, can be interpreted as taking a length scale as Δ and velocity scale as $q = \Delta|S|$ (here, C_s is a Smagorinsky constant, $|S| = \left(S_{ij}S_{ij}\right)^{0.5}$, and S_{ij}). Using a velocity scale ratio $\beta = q/C$, the exponent of d can be estimated as

$$\ln(d) = \frac{2\pi Im(\omega(k))}{k|v_p|} = -\frac{2\pi \nu_{SGS} k}{|v_p|} = -\frac{2\pi C_s^2 \beta (k\Delta)^2}{\sin\,(k\Delta)} \tag{6}$$

Here, v_p of the Linear is used by assuming the advection term is discretized as well. The range of β is unknow, but it represents the ratio between the SGS velocity scale q and advection velocity C. When we consider the contributions of the SGS turbulent kinetic energy (TKE), TKE, and mean kinetic energy (MKE) to total kinetic energy, their magnitude can generally be ordered as SGS-TKE<TKE<MKE. Therefore, β can be smaller than 1.0, although it is only a rough approximation. Due to page limitations, the results for d will be explained in the presentation.

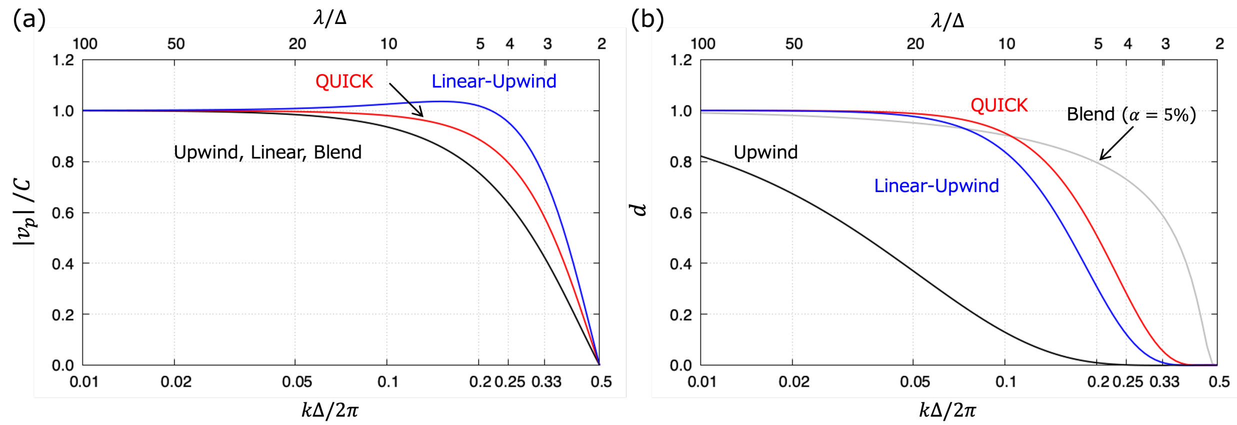

Figure 1: Effects of advection schemes on (a) normalized phase velocity v_p/C and (b) decay factor exp $(-d)$ as a function of normalized wave number $k\Delta/2\pi$ and wavelength λ/Δ.

4. CONCLUSIONS

The effect of advection schemes and SGS models on LES results was theoretically discussed by introducing dispersion relation, phase velocity, and decay factor in wave propagation theory. To provide more intuitive and reliable best practice guidelines for selecting suitable numerical conditions, especially for practitioner of numerical simulation, this work contributes to deepen understanding for CFD specialists for near future establishment of LES guidelines.

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