

## Robust verification for computational wind engineering applications

Radostin Mitkov<sup>a</sup>, Petar O. Hristov<sup>a</sup>, Franziska Hunger<sup>b</sup>, Andreas Mark<sup>b</sup>

<sup>a</sup>GATE Institute, Sofia, Bulgaria

{ radostin.mitkov, petar.hristov }@gate-ai.eu

<sup>b</sup>Fraunhofer-Chalmers Centre for Industrial Mathematics, Sweden

{ franziska.hunger, andreas.mark }@fcc.chalmers.se

### Summary

This study presents an application of the robust verification method (RVM) for numerical uncertainty estimation in a CWE context and outlines the challenges associated with it. The reliability of the RVM estimates is analysed and compared to that of results from approaches in current practice and a discussion on optimal RVM settings for CWE is provided. A RANS pedestrian wind study in a real-world urban district is used as a test ground. In addition, the paper utilises the RVM to develop a joint estimator for discretisation and iterative convergence uncertainty. A discussion and example on the practical subsequent use of the estimator is presented using a set of wind-tunnel measurements for the modelled region. A way to assess the level of calibration of the final estimators and a guide to the interpretation of their values is also provided, aiming to help computational wind engineers in the development of reliable models.

**Keywords:** *CFD, CWE, verification, numerical uncertainty, urban wind flow, robust verification analysis*

## 1 MOTIVATION

Similarly to other numerical methods, CWE simulations require verification, validation, and uncertainty quantification (VVUQ) efforts to demonstrate the credibility of the obtained results. Verification is the process of determining the quality of the solution of a computational implementation for a mathematical model. In particular, solution verification is concerned with estimating the numerical uncertainty of that implementation when applied in a problem of practical interest.

As a continuation of the paper by Mitkov et al. (2025), which explored different methods for verification in CWE, and motivated by the work of Rider et al. (2016), in this paper we ask several questions about practical CWE verification, the reliability of the estimated numerical uncertainty, what it captures and whether the current interpretations for it are adequate.

We frame the problem of solutions verification as follows. Let  $T$  be the true value of the quantity we are interested in. In the case of solution verification,  $T$  is not the true value of the physical process we are trying to model, but is the exact solution of the algebraic equation implemented in the numerical model, attained when spatial, temporal or any other discretisation used in the model reaches levels at which the size of each discrete element (step, iteration) approaches 0. We cannot generally observe  $T$ .

The analysed model can be run at various levels of discretisation,  $\ell$ , to try and estimate  $T$ . The resulting quantity is denoted with  $E_\ell$ . We denote the finest available estimation as  $E_N$ . In general, as  $\ell$  increases for well-behaved models,  $E_\ell \rightarrow T$ , which is the behaviour tested through classical convergence studies.

To try and get a more complete estimate of  $T$ , with all the available data, one can compute some statistic, such as a mean or a median, of  $T$ , based on the set  $\mathcal{E} = \{E_1, \dots, E_N\}$ . This can be done conveniently through some form of an error expression. A popular choice is to represent this error with a power law of the form

$$E_\ell = T + Ch_\ell^p \equiv T + Cn_\ell^{-p} \tag{1}$$

## 2 ROBUST VERIFICATION ANALYSIS

The robust verification analysis proposed by Rider et al. (2016), to complement the family of regression approaches to solution verification (e.g., by Eça and Hoekstra (2014)), employs Eq. (1) to model the discretisation error in the solution. The procedure aims to solve the following multiple constraint minimization problems

$$\begin{aligned} & \arg \min_{\ell} \|E_{\ell} - T - Ch_{\ell}^p\|_{\beta} \\ & \text{subject to } p_{\text{lower}} \leq p \leq p_{\text{upper}}, T_{\text{lower}} \leq T \leq T_{\text{upper}} \end{aligned} \quad (2)$$

where  $\beta$  is the optimization problem metric, such as  $L_1$ ,  $L_2$ , or a regularised version of one.

The optimization procedure goes through several iterations taking into account different data sets of grid solutions. Each data set is defined as  $S_j = [(h_{N-j}, E_{N-j}), \dots, (h_N, E_N)]$  with the first set consisting of the two finest grids:  $S_1 = [(h_{N-1}, E_{N-1}), (h_N, E_N)]$ . Each consecutive data set adds the next finest grid to the set, creating a result more heavily influenced by the finer grids, which are assumed to achieve more accurate results. Several estimates of  $T$  are then obtained and the median value,  $T_{med}$ , is computed along with the median deviation,  $\Sigma_{med} = \text{median}\|T_i - T_{med}\|$ . The mesh converged solution is then estimated to belong to a 95% confidence interval  $T_{med} \pm 3\Sigma_{med}$ . Finally, the numerical uncertainty is given as follows:

$$U_{num} = |T_{median} - E_N| + 3\Sigma_{median} \quad (3)$$

## 3 CFD SIMULATION: COMPUTATIONAL MODEL, DOMAIN, GRID AND SOLVER PARAMETERS

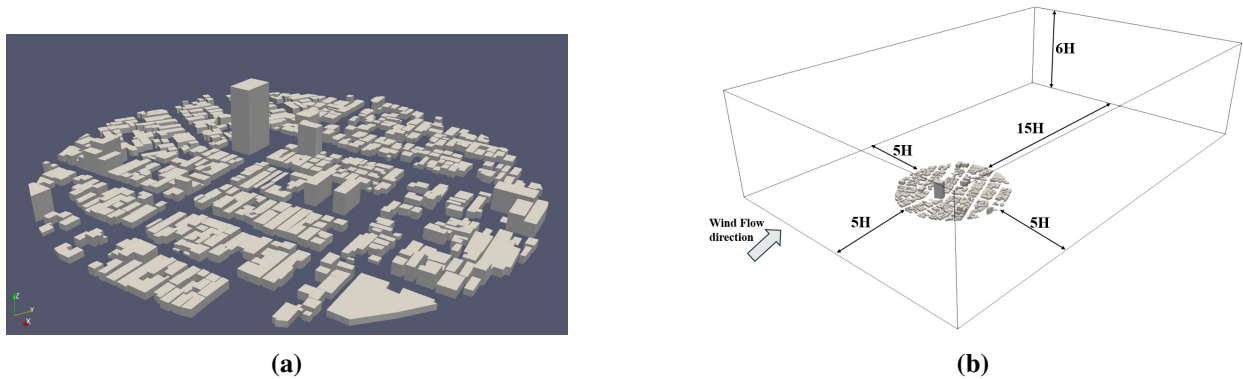
To demonstrate the applicability of the robust verification analysis to a real-world problem, the wind flow around an actual building complex was simulated. The geometry, seen in Figure 1(a), is provided by the Architectural Institute of Japan (AIJ) Database<sup>1</sup>. The steady-state Immersed Boundary Octree Flow solver, IBOFlow<sup>®</sup> which utilizes the mirroring immersed boundary method with a structured adaptive octree background grid, was used (Mark et al. 2011). The third-order bounded QUICK scheme and the second order central differencing scheme was used respectively for the convective and diffusive terms. The computational domain, shown in Figure 1(b), was created following the best practice guidelines by Tominaga et al. (2008). Westerly wind direction was used with the inlet velocity and turbulent properties profiles taken by AIJ (2016).

Five different grids G1 to G5 were employed in this study, with G1 being the coarsest and G5 the finest one, as presented in Table 1. The domain is discretised using a base grid with six refinement levels around the buildings with a ratio of 1:2. Subsequent grids were refined with a factor of 1.225 in each dimension. A common verification grid, consisting of a horizontal plane at pedestrian height (2 meters above ground), was used to map the quantities of interest from each grid to common nodes.

## 4 RESULTS, DISCUSSION AND CONCLUSIONS

The robust verification workflow can be completed for any convergent or divergent datasets even though it may not produce improved results for the divergent cases. For the purposes of the current

<sup>1</sup>Available at [https://www.aij.or.jp/jpn/publish/cfdguide/index\\_e.htm](https://www.aij.or.jp/jpn/publish/cfdguide/index_e.htm)



**Figure 1:** (a) Model geometry; (b) computational domain.  $H$  denotes the height of the tallest building.

**Table 1:** Specification summary of the five grids used in this study

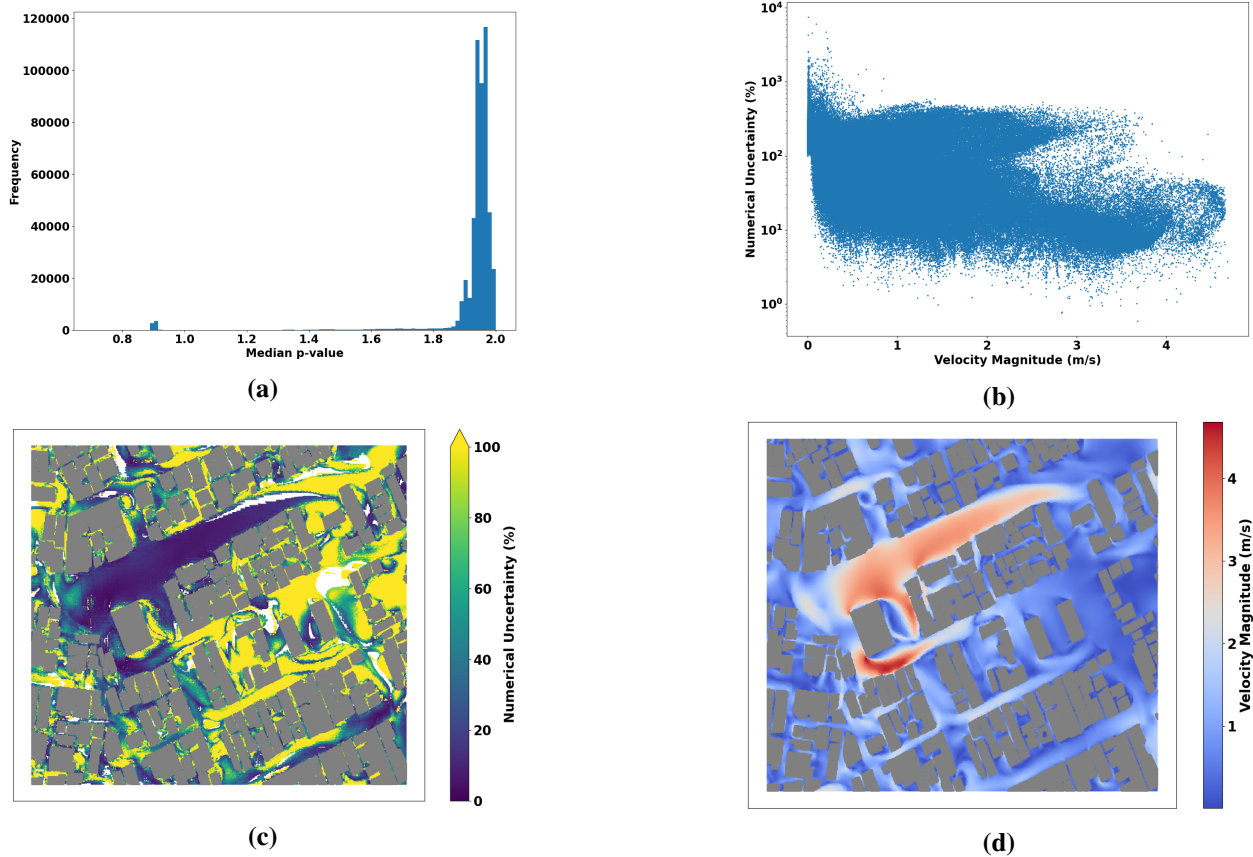
Grid	Base grid cell size [m]	Minimum grid size [m]	Total number of cells
G1	27.00	0.4219	$7.78 \times 10^6$
G2	22.05	0.3445	$12.78 \times 10^6$
G3	18.00	0.2813	$20.54 \times 10^6$
G4	14.70	0.2297	$34.12 \times 10^6$
G5	12.00	0.1875	$61.50 \times 10^6$

study,  $p = [0.5, 3.5]$  and  $T = [0, 10]$ . Figure 2(a) presents a histogram of the median values of  $p$  for all points in the verification grid. As we are employing the QUICK scheme, which despite being a mixed-order scheme, is expected to converge in a second-order manner. This is evidenced by the histogram where virtually all  $p$ -values fall in the range between 1.9 and 2.

For all but oscillatory converging nodes with non-converging absolute differences between the solutions, the numerical uncertainty was computed and is presented in Figure 2(c). It can be observed that the areas with lower numerical uncertainties mostly correspond to areas with higher velocities around the tall building (see Figure 2(d)). Additionally, areas in the wake region and with lower velocity magnitudes, exhibit larger numerical uncertainties. Figure 2(b) supports this observation and reveals that nodes with higher velocity magnitudes mostly have more concentrated and comparatively lower numerical uncertainties, compared to nodes with lower velocities, which exhibit larger spread. We owe this observation to the fact that RANS simulations are better capable of predicting the high-velocity regions which are dominated by the mean component of the flow, and investigate this further in the full paper.

## ACKNOWLEDGEMENTS

This research is part of the GATE project funded by the Horizon 2020 WIDESPREAD-2018-2020 TEAMING Phase 2 programme under grant agreement no. 857155 and the programme Research, Innovation and Digitalization for Smart Transformation 2021-2027 (PRIDST) under grant agreement no. BG16RFPR002-1.014-0010-C01. The work was also supported by the Swiss National Science Foundation and the Bulgarian Ministry of Education and Science under Grant Agreement No. IZPYZO\_228861 for the project “Automatic maximally rigorous uncertainty quantification to enable design for certification” (unCertify)



**Figure 2:** (a) Histogram of median p-values; (c) Spatial distribution of numerical uncertainty in percentages; (d) Velocity field plot with westerly wind; (b) Velocity magnitude against numerical uncertainty plot.

## REFERENCES

- AIJ. AIJ Benchmarks for Validation of CFD Simulations Applied to Pedestrian Wind Environment around Buildings. Technical report, Architectural Institute of Japan (AIJ), Tokyo, Japan, 2016.
- L. Eça and M. Hoekstra. A procedure for the estimation of the numerical uncertainty of CFD calculations based on grid refinement studies. *J. Comp. Phys.*, 262:104–130, 2014.
- A. Mark, R. Rundqvist, and F. Edelvik. Comparison between different immersed boundary conditions for simulation of complex fluid flows. *Fluid Dyn. and Mat. Proc.*, 7:241–258, 2011.
- R. Mitkov, P. O. Hristov, F. Hunger, and A. Mark. On the variety and adequacy of different solution verification approaches in computational wind engineering. *J. Phys.: Conf. Series*, 2025.
- W. Rider, W. Witkowski, J. R. Kamm, and T. Wildey. Robust verification analysis. *Journal of Computational Physics*, 307:146–163, 2016.
- Y. Tominaga, A. Mochida, R. Yoshie, H. Kataoka, T. Nozu, M. Yoshikawa, and T. Shirasawa. AIJ guidelines for practical applications of CFD to pedestrian wind environment around buildings. *Journal of Wind Engineering and Industrial Aerodynamics*, 96(10-11):1749–1761, 2008.