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Assessment of regularized variational multiscale eddy-viscosity models in homogeneous isotropic turbulence

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SUMMARY (10 PT)

Direct numerical simulation (DNS) of turbulent flows at high Reynolds numbers remains computationally prohibitive, motivating the use of large-eddy simulation (LES). The accuracy of LES depends on subgrid-scale (SGS) models, yet classical Smagorinsky eddy-viscosity formulations often exhibit excessive dissipation. This work investigates the mechanisms of SGS models, with a particular focus on both classical and regularized variational multiscale (RVM) eddy-viscosity approaches. Detailed analyses and results are presented in the full paper.

Keywords: *Large-Eddy Simulation, SGS models, Homogeneous Isotropic Turbulence, Pseudo-spectral method*

1. INSTRUCTION

Although computational power has been increasing now at an ever-increasing speed, direct numerical simulations (DNS) associated with high Reynolds numbers or large domains (e.g., Caprace et al., 2020) still require a high computational cost beyond the budget. This triggers the need for the Large Eddy Simulation (LES), the technique that has been developed since 1960's (e.g., Smagorinsky 1963). The idea of LES is to simulate the large-scale motion of the flow while the small-scale turbulence effects are modeled. Thus, the LES can be carried out on a coarser grid and, hence, reduces the computation cost to resolve the small-scale flow. The effects of turbulence of scales smaller than the grid size are transformed into the stress tensor and moved to the right-hand side of the Navier-Stokes equation as a source term, which is usually called the sub-grid scale (SGS) stress.

The subgrid-scale (SGS) stress is commonly modeled using the Boussinesq hypothesis, yielding an eddy-viscosity formulation analogous to molecular diffusion but with an effective viscosity ν_e:

$$\tau_{ij}^{(M)} = -2\nu_e \bar{S}_{ij} \tag{1}$$

where $\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$ is the resolved strain-rate tensor. Owing to its simplicity, the eddy-viscosity model is widely used in LES. For bluff-body-induced separated flows (e.g., Travin et al., 1999; Wornom et al., 2011; Moussaed et al., 2014), validation typically relies on comparisons of mean and fluctuating velocity and pressure statistics. While such comparisons are necessary, they provide limited insight into the underlying model mechanisms and their influence on time-averaged flow statistics. Therefore, it is best to understand the fundamentals of the SGS models, which is the objective of this work. To investigate subgrid-scale effects on coarse grids, energy across different flow length scales is analyzed using the Fourier transform. Accordingly, this work employs the Fourier pseudo-spectral method for numerical simulation (e.g., Canuto et al., 1988; Peyret, 2002). Freely decaying three-dimensional homogeneous isotropic turbulence (HIT) is adopted as a canonical benchmark flow. Sec. 2 presents the theoretical background of the Fourier pseudo-spectral method. Simulation results are discussed in Sec. 3.

2. THE GOVERNING EQUATIONS

The evolution of homogeneous isotropic turbulence (HIT) is simulated using a Fourier pseudo-spectral method. The resolved velocity $\bar{\boldsymbol{u}}(\boldsymbol{x},t)$ is expanded in Fourier modes,

$$\bar{\boldsymbol{u}}(\boldsymbol{x},t) = \sum_{\boldsymbol{k}} \bar{\boldsymbol{u}}(\boldsymbol{k},t)\exp(\underline{i}\boldsymbol{k}\cdot\boldsymbol{x}) \tag{2}$$

which allows spatial derivatives to be computed efficiently in spectral space, with $\partial/\partial x_j \to \underline{i}k_j$ and $\nabla^2 \to -k^2$. For a triply periodic domain of size (L_1, L_2, L_3) discretized on (N_1, N_2, N_3) grid points, the wavenumbers are $k_i = n_i(\Delta k_i)$, where n_i and Δk_i are referred to as the wave-integer and wavenumber increment.

Applying the same decomposition to pressure, nonlinear advection, and SGS terms yields the LES momentum equation in spectral space,

$$\frac{\partial \bar{u}_i(\boldsymbol{k})}{\partial t} + \bar{G}_i(\boldsymbol{k}) = -\frac{1}{\rho}\underline{i}k_i\bar{p}(\boldsymbol{k}) - \nu k^2\bar{u}_i(\boldsymbol{k}) - \bar{g}_i(\boldsymbol{k}) \tag{3}$$

Nonlinear terms are evaluated using the 3/2 de-aliasing rule. In this framework, the LES filter is implemented as a sharp spectral low-pass filter, and no explicit spatial filtering is applied. Two filters are considered: cubical truncation, applied independently in each coordinate direction, and spherical truncation, applied radially with respect to $k = \sqrt{k_1^2 + k_2^2 + k_3^2}$.

In the pseudo-spectral framework, the velocity energy spectrum is readily obtained as

$$\bar{E}(\boldsymbol{k}) \equiv \bar{u}_j^{(c)}(\boldsymbol{k})\bar{u}_j(\boldsymbol{k})/2 \tag{4}$$

where $(\cdot)^{(c)}$ denotes the complex conjugate. Using the spectral momentum equation Eq. (3), the temporal evolution of the energy spectrum is

$$\frac{\partial \bar{E}(\boldsymbol{k})}{\partial t} = \frac{1}{2}\left(\bar{u}_j(\boldsymbol{k})\bar{F}_j^{(c)}(\boldsymbol{k}) + \bar{u}_j^{(c)}(\boldsymbol{k})\bar{F}_j(\boldsymbol{k})\right) \tag{5}$$

with the forcing term

$$\bar{F}_i(\boldsymbol{k}) = -\left(\bar{G}_i(\boldsymbol{k}) + \frac{\underline{i}k_i}{\rho}\bar{p}(\boldsymbol{k})\right) - \nu k^2\bar{u}_i(\boldsymbol{k}) - \bar{g}_i(\boldsymbol{k}) \tag{6}$$

The nonlinear term represents spectral energy transfer, the viscous term accounts for molecular dissipation, and the SGS term $\bar{g}_i(\boldsymbol{k})$ corresponds to subgrid-scale dissipation, which is central to LES and discussed further.

3. RESULTS

For current applications in 3D HIT, a cubical domain of length $L_1 = L_2 = L_3 = L$ and node number $N_1 = N_2 = N_3 = N$ results a wavenumber increment $\Delta k_1 = \Delta k_2 = \Delta k_3 = \Delta k = 2\pi/L$. Uniform grid is used for the current Fourier pseudo spectral method, so the flow is assumed to be periodic in all directions. For all simulations, the domain length is $L = 8\pi$(m), and the initial kinetic energy, $E_0 = \langle u_i u_i\rangle/2 = 0.15$ (m^2/s^2), where $\langle \quad \rangle$ denotes the spatial averaging. The time step size is $\Delta t = 0.002$(sec). The simulated duration is $T = 15$. Explicit second-order Runge-Kutta time advancement technique is used. The parameters of numerical simulations are summarized in Table 1.

Finer grids of sizes 257^3 and 337^3 are used for DNS simulations. Coarser grid of sizes 49^3 and 129^3 are used for LES simulations. Two initial (nominal) integral length of the HIT are considered, i.e., $l_o = 2$(m) & 6(m). Three kinematic viscosity, i.e., $\nu = 0.005\ (m^2/s)$, $0.002\ (m^2/s)$, and $0.001\ (m^2/s)$ are considered. Three models of SGS stress are considered. The first one is the exact SGS stress obtained from the truncated DNS. The second one is the classic Smagorinsky (SMAG) model, which is of 'All-All' (AA) type of the model. The third one is the Regularized Variational Multiscale (RVM) model, in which the 'All-Small' (AS) and 'Small-Small' (SS) models are considered. To evaluate the coefficient for the eddy viscosity, both static and dynamic methods are considered. In the static method, the exact global dissipation is first calculated from the exact SGS using $\langle \bar{\varepsilon}_\tau \rangle = \langle \tau_{ij} \bar{S}_{ij} \rangle$, and compared to the global dissipation from the models, at time $T = 2$. The appropriate model constant is selected so as to impose the same amount of dissipation of the exact value.

Table 1: DNS and LES parameters considered

Name	Value
Cubical domain size, L (m)	8π
Time step increment, Δt (sec)	0.002
DNS grid size	$257^3, 337^3$
LES grid size	$49^3, 129^3$
Nominal integral length scale, l_o (m)	2, 6
Kinematic viscosity, ν (m^2/s)	0.005, 0.002, 0.001
Sharp spectral filter	Cubical, Spherical
SGS stress model	Exact, SMAG (AA), RVM (AS, SS)
Eddy viscosity coefficient method	Static (G), Dynamic (D)

Spherical truncation is applied, and static model coefficients are calibrated by matching the modeled dissipation to the exact value at $t = 2(s)$, after the initial HIT transient. Energy histories are shown in Fig. 1. In most cases, the classical SMAG (AA) model underpredicts global energy at later times due to its well-known over-dissipation, whereas the AS and SS models yield closer agreement with exact LES. An exception occurs for the most challenging case ($\nu = 0.001(m^2/s)$), where the exact LES is derived from an under-resolved DNS ($N = 337$).

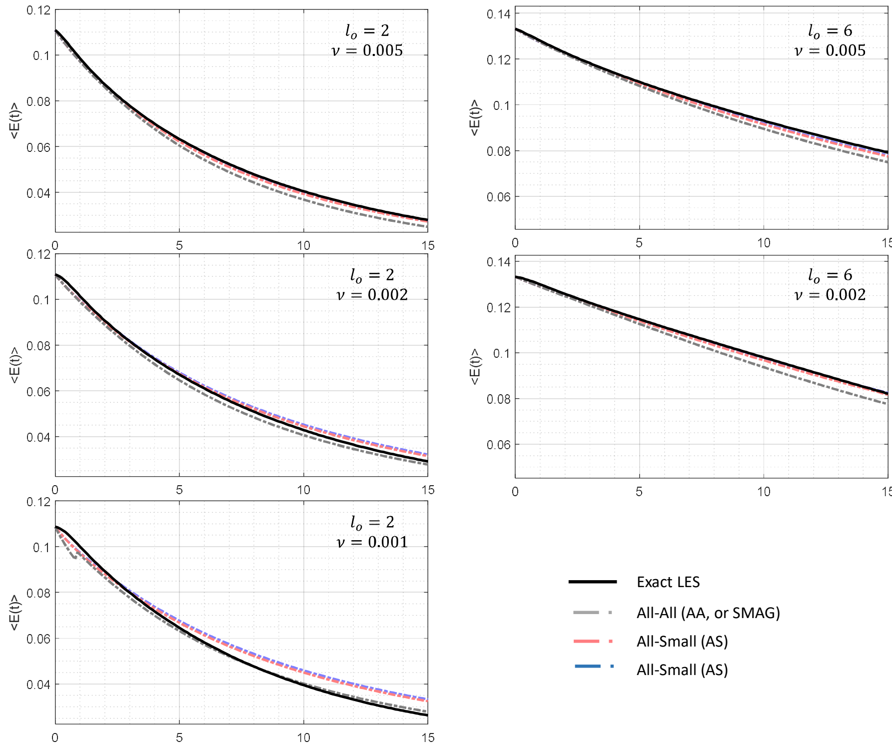

Fig. 1: Total energy history, E(t), of exact LES's and model LES's of N = 49 using spherical truncation and static coefficients in the RVM eddy viscosity models All-All (AA, or SMAG), All-Small (AS), and Small-Small (SS).

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