

# Probabilistic optimization of twin box shape and steel plate size of suspension bridges under flutter constraint

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## SUMMARY

A reliability-based design optimization (RBDO) study was performed for a suspension bridge deck with a twin-box girder, under a probabilistic flutter constraint. A Surrogate model based on computational fluid dynamics simulations was constructed to predict the flutter derivatives as a function of reduced velocity over the entire design domain. The twin-box cross-section was parametrized by three design variables: the width of each box, the girder depth, and the gap between the two boxes. Uncertainties in both the flutter derivatives and the wind speed were considered. The optimization sought the deck geometry and plate thicknesses of the twin box that minimize material usage while satisfying a probabilistic flutter constraint and other deterministic structural constraints.

**Keywords:** *reliability-based design optimization, CFD, twin-box girder, surrogate model, flutter derivatives, uncertainty*

## 1. RELIABILITY-BASED DESIGN OPTIMIZATION AND TWIN-BOX DESIGN

The design of long-span suspension bridges requires a comprehensive assessment of structural performance, particularly with respect to wind-induced vibrations such as flutter. Among the various design parameters, the girder geometry and the thickness of the steel plates that form the box section play a critical role in determining the aeroelastic behavior. In recent years, twin-box girders have gained prominence in long-span bridge design due to their superior flutter resistance compared to mono-box configurations. Their aeroelastic response, however, is highly sensitive to the gap width between the two boxes.

Structural optimization is essential for reducing material usage, which directly influences overall construction costs. Minimizing steel consumption is also important from an environmental perspective, as steel production is a major contributor to global greenhouse gas emissions. Reliability-based design optimization (RBDO) incorporates uncertainties into the design process, offering a more robust and dependable framework than traditional deterministic approaches. Previous studies have successfully applied RBDO to optimize the size and shape of single box-girder decks (Kusano et al., 2014, 2020). To extend this methodology to twin-box configurations, an extensive research program was carried out at the University of Coruña, involving numerical forced-vibration simulations to obtain flutter derivatives, complemented by experimental validation for a range of cross-sectional designs (Badhurshah et al., 2023).

In this study, RBDO is applied to the geometry and steel plate thicknesses of a twin-box girder for a suspension bridge, accounting for uncertainties in both the flutter derivatives and the wind speed.

The objective is to minimize the material weight of the twin-box girder while satisfying aeroelastic and structural performance requirements.

To define a specific section of the twin-box girder, three design variables were selected: the width (C) and depth (D) of each box section and the gap (D) between them, as shown in Figure 1. The section is symmetrical about the central vertical axis. The width (C) can range  $\pm 5\%$  of the nominal value of 14 m, while the depth (D) can vary from  $-10\%$  to  $+25\%$  of the nominal value of 2 m. The gap ranges from  $-25\%$  to  $60\%$  of the nominal value of 8 m.

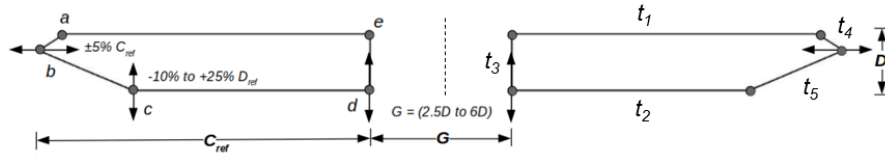


Figure 1: Definition of the twin-box section with 3 design variables

A Kriging surrogate model was built to predict the flutter derivatives as a function of reduced velocity across the full design domain during the optimization process. This model was trained with 75 forced vibration CFD simulations to extract the flutter derivatives of 15 design configurations at 5 reduced velocity values. The input variables for the surrogate models—width (C), depth (D), and gap (G)—were nondimensionalized, and the models were then used to predict the flutter derivatives across a range of reduced velocities.

## 2. RELIABILITY ANALYSIS AND RBDO OF SUSPENSION BRIDGES

Reliability analyses provide the probability of failure concerning structural limit states while accounting for system uncertainties. In this study, uncertainties in the flutter derivatives obtained from the surrogate models based on the CFD simulations are considered as well as those in the extreme wind velocity at the bridge location.

Each flutter derivative value at a given reduced velocity is assumed to follow a normal distribution, with the computed value as the mean value. We adapted a linearly increasing standard deviation to model increasing uncertainty with larger wind velocity. This standard deviation was set to 0 at  $V^*=0$  and  $\sigma_{max}=0.15\mu$  at  $V^*=30$  (Baldomir et al., 2013).

The limit state function that defines the structural failure under flutter can be written as:

$$G(\mathbf{x}) = V_f(x_i) - x_w \quad i = 1, \dots, n \quad (1)$$

where  $V_f$  is the flutter wind speed,  $x_i$  are  $n$  random variables representing a set of points of flutter derivatives and  $x_w$  is the random variable of the extreme wind velocity at the bridge location. The total number of random variables is expressed as vector  $\mathbf{x}$  of  $n+1$  dimension.

The First Order Reliability Method (FORM) was used in this study to compute the reliability index,  $\beta$  of the bridge structure due to flutter failure. This method calculates the minimum distance from the mean value of the random variables to the failure surface  $G(\mathbf{U})=0$  using the Hasofer-Lind method (Hasofer and Lind, 1974).

A reliability-based design optimization (RBDO) was conducted on a long-span suspension bridge with characteristics similar to the Great Belt East Bridge. The baseline geometry of the deck section is shown in Figure 1. A 3D finite element model using Abaqus was employed to perform both static and modal analysis of the bridge during the optimization process. The initial design's natural frequencies and mode shapes were calculated, and several relevant modes were selected for flutter analysis. The random variables and the limit state function were defined earlier in Section 3. The mean and standard deviation of the extreme wind speed were based on the Danish Wind Code DS410 (1998).

The RBDO for shape optimization was performed considering three design variables: the depth (D), width (C), and gap (G) of the box section. The objective was to minimize the girder weight of the twin-box.

The RBDO problem is formulated as follows:

Minimize: *Girder weight (d)*

subject to:

$$g_1 : P_f [V_f(\mathbf{x}) - x_w \leq 0] \leq P_f \quad P_f = \Phi(-\beta) \text{ where } \beta=4.0 \quad (2)$$

$$g_2 : -5\% \leq \Delta C \leq 5\%; \quad -10\% \leq \Delta D \leq 10\%; \quad -25\% \leq \Delta G \leq 60\% \quad (3)$$

$$g_3 : z_d / z_{\max} - 1 \leq 0 \text{ where } z_{\max}=L/500; \quad L=1624 \text{ m} \quad (4)$$

$$g_4 : \sigma_c = 565 \text{ MPa} \quad (5)$$

$g_1$  is the probabilistic flutter constraint, while  $g_2$  sets the design limits.  $g_4$  and  $g_5$  are deterministic constraints restricting the maximum girder vertical displacement and the maximum cable stress obtained from static analyses.

The flowchart of the optimization process is shown in Figure 2.

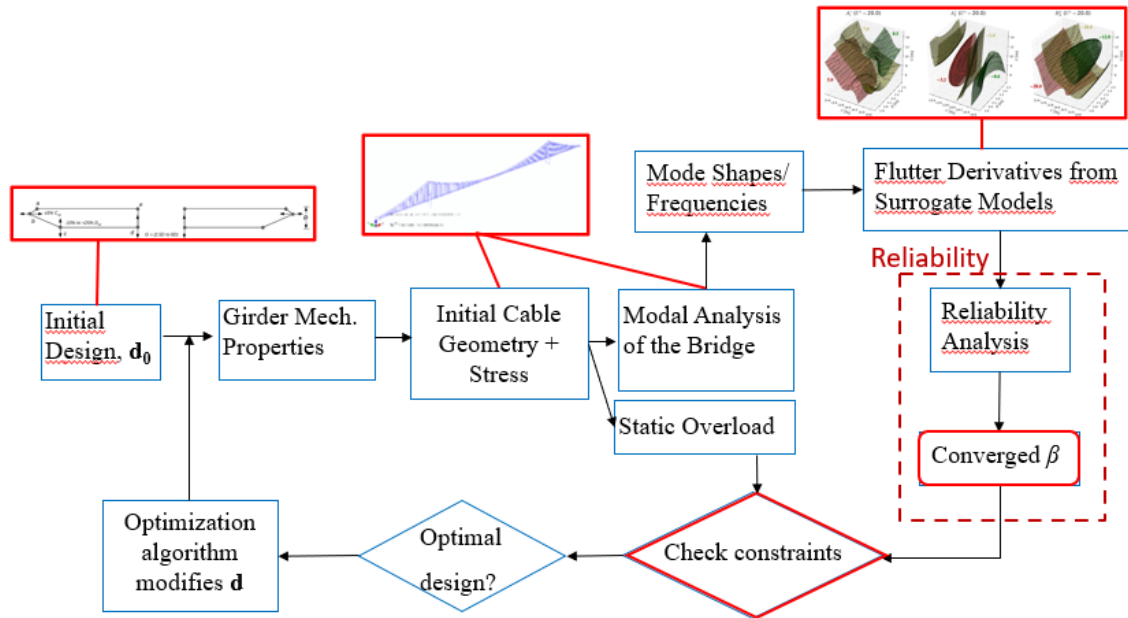


Figure 2: Flowchart of the RBDO applied to the twin-box

### 3. RBDO RESULTS

A result of the RBDO for the target  $\beta = 7.0$  is shown in Figure 4. The initial design was  $[C, D, G] = [14.0, 2.5, 5.0]$ , and the initial steel plate size was 25.0 mm on all sides. In this case, the shape variables did not change much, but the size variables of the steel plate decreased as much as 14% and the objective function was reduced by 9.5%.

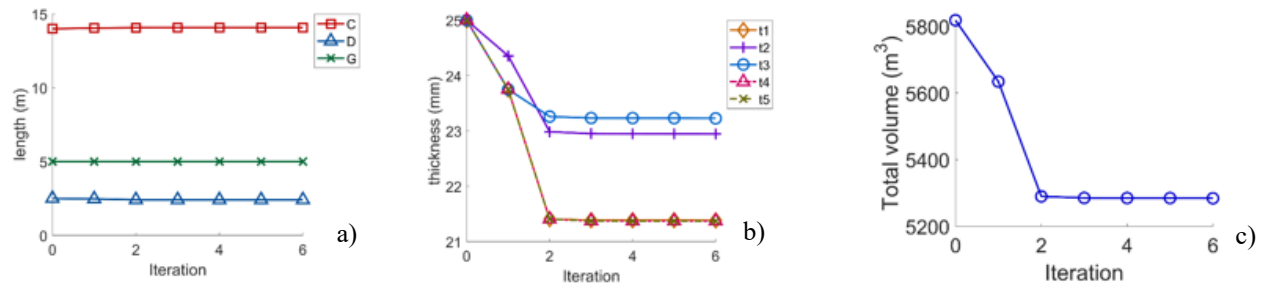


Figure 3: Evolution of variables: a) shape parameters b) size variables c) objective function

### 4. CONCLUSIONS

The probabilistic optimization was carried out for the cross-sectional shape and steel plate thicknesses of a twin-box deck for a long-span suspension bridge, accounting for uncertainties in both the extreme wind velocity and the flutter derivatives predicted by surrogate models based on CFD simulations. This approach enables an optimal design that explicitly incorporates the uncertainties associated with flutter phenomena, resulting in a solution that is more robust and more accurate than conventional deterministic methods. The outcome is a reduction in steel usage, contributing to more sustainable and environmentally responsible bridge design.

### ACKNOWLEDGEMENTS

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